Course Specifications

Course Title:	**Measure Theory and Integration**
Course Code:	**30114105-3**
Program:	**BSc. Mathematics 301100**
Department:	**Mathematics**
College:	**Al Leith University College**
Institution:	**Umm Al Qura University**

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A. Course Identification

1. Credit hours:	4 hours
2. Course type	
a.	University ☐ College ☐ Department ☑ Others ☐
b.	Required ☑ Elective ☐
3. Level/year at which this course is offered:	Eighth Level / Fourth year
4. Pre-requisites for this course (if any)**:**	Real Analysis (2) 30113103-3
5. Co-requisites for this course (if any)**:**	None

6. Mode of Instruction (mark all that apply)

No	Mode of Instruction	Contact Hours	Percentage
1	**Traditional classroom**	(3 hours) x (15 weeks)	100%
2	**Blended**	0	0%
3	**E-learning**	0	0%
4	**Correspondence**	0	0%
5	**Other**	0	0%

7. Actual Learning Hours (based on academic semester)

No	Activity	Learning Hours
Contact Hours		
1	**Lecture**	(3 hours) x (15 weeks)
2	**Laboratory/Studio**	0
3	**Tutorial**	0
4	**Others** (Exam)	8 hours
	Total	53 hours
Other Learning Hours*		
1	**Study**	45 hours
2	**Assignments**	25 hours
3	**Library**	15 hours
4	**Projects/Research Essays/Theses**	15 hours
5	**Others** (specify)	0
	Total	100 hours

* The length of time that a learner takes to complete learning activities that lead to achievement of course learning outcomes, such as study time, homework assignments, projects, preparing presentations, library times

B. Course Objectives and Learning Outcomes

1. Course Description

The program of the course contains as described below.

Rings, algebras, σ-rings, σ-algebras, monotone families, generated families. Intervals, the generated ring P and algebra Φ in P d . Open, closed and Borel sets in metric spaces. The definition and properties of non-negative measures on a σ-rings (rings) of subsets of a given set. Finite and σ-finite measures. Generalized distribution functions and a construction of measures on P and Φ. The definition and properties of outer measures. The construction of the outer measure induced by a given measure on a ring. The theorems (with proofs) of Caratheodory, on extending a measure from a ring (algebra) to the generated σ-ring (σ-algebra) and on uniqueness of the extension. Complete measures and a construction of the completion. The definition, properties and characterizations of the Lebesgue measure (measurable sets) in P d . Nonmeasurable sets. Borel and Lebesgue measurable functions and their properties. Nonmeasurable functions. The convergences almost everywhere and in measure. The theorems of Egoroff, Luzin and Riesz. The definition and properties of the integral with respect to a nonnegative measure (the Lebesgue integral); real- and complex-valued integrable functions; basic convergence theorems (Lebesgue, Fatou). Comparison of the proper and improper Lebesgue and Riemann integrals. The spaces L p and integral inequalities (Schwarz, Hölder, Minkowski). The definitions and properties of real-valued, complex valued measures and signed measures, the variation of a measure, the Hahn and Jordan decompositions. Absolute continuity and singularity of measures.

2. Course Main Objective

The aim of the course is to provide for the students an important extension of the theory of measure and integration, shortly presented in the course Real Analysis. The students are expected to understand mathematical notions as well as to use them in practice.

3. Course Learning Outcomes

	CLOs	Aligned PLOs
1	**Knowledge:**	
1.1	Present the measurability concept for functions and sets	K1
1.2	Recognize the -almost everywhere- notion	K3
1.3	Describe the notion of measurable functions	K3
1.4	State the main properties of Lebesgue integrals	K4
2	**Skills:**	
2.1	Apply Hölder's and Minkowski's inequalities	S5
2.2	Illustrate an account of the construction of the Lebesgue integral	S4
2.3	Use properly Lebesgue monotone and dominated convergence theorems	S3
2.4	Connect Riemann integral and Lebesgue integral	S1
3	**Competence:**	
3.1	Develop probabilistic concepts (random variables, expectation and limits) within the framework of measure theory	C2
3.2	Employ a wide range of references and critical thinking	C5
3.3	Generalize mathematical concepts in problem-solving through integration of new material and modeling	C4

C. Course Content

No	List of Topics	Contact Hours
1	Preliminaries on Set Theory and Topology: Basic concepts from set theory and topology. Rings, sigma-rings, algebras and sigma-algebras of sets. Generated set systems, connection between some types of generated systems, examples of rings and sigma-rings over intervals. Concept of set function and some types of set functions. Properties of additive functions defined on rings of sets.	9
2	Measure: Non-negative sigma-additive functions and their properties, measure, connection of measure with non-negative additive functions, examples of measures over intervals. Outer measure, measurability in sense of Carathéodory. Extension and completion of measure, m*-measurability and completeness, system of m*-measurable sets with respect to induced measure. Existence and uniqueness of extension of a measure. Lebesgue measure	12
3	Measurable functions: Measurable space, simple measurable functions, measurable functions and criteria of measurability of functions. Further properties of measurable functions, sequences of measurable functions, Borel and Lebesgue measurability.	12
4	Integral: Measure space, integral of simple functions. Integral of a non-negative measurable function, definition of integral of a measurable function and its properties. Integral and limit of a sequence of functions, integral as a set function, Lebesgue and Lebesgue-Stieltjes integral, convergence theorems.	12
Total		45

D. Teaching and Assessment

1. Alignment of Course Learning Outcomes with Teaching Strategies and Assessment Methods

Code	Course Learning Outcomes	Teaching Strategies	Assessment Methods
1.0	**Knowledge**		
1.1	Present the measurability concept for functions and sets	Lecture Tutorials	Exams (Quizzes, Midterm and Final). Written and possibly oral exam at the end of the course. In addition, compulsory work may be given during the course
1.2	Recognize the -almost everywhere- notion		
1.3	Describe the notion of measurable functions		
1.4	State the main properties of Lebesgue integrals		
2.0	**Skills**		
2.1	Apply Hölder's and Minkowski's inequalities	Lecture	Exams (Quizzes, Midterm and Final).

Code	Course Learning Outcomes	Teaching Strategies	Assessment Methods
2.2	Illustrate an account of the construction of the Lebesgue integral	Individual or group work	Homework
2.3	Use properly Lebesgue monotone and dominated convergence theorems		
2.4	Connect Riemann integral and Lebesgue integral		
3.0	**Competence**		
3.1	Develop probabilistic concepts (random variables, expectation and limits) within the framework of measure theory	Lecture Individual or group work	Exams (Quizzes, Midterm and Final). Research Essays
3.2	Employ a wide range of references and critical thinking		
3.3	Generalize mathematical concepts in problem-solving through integration of new material and modeling		

2. Assessment Tasks for Students

#	Assessment task*	Week Due	Percentage of Total Assessment Score
1	Midterm Test (1)	6^{th} week	20%
2	Midterm Test (2)	12^{th} week	20%
3	Homework + Reports + Quizzes	During the semester	10%
4	Final Examination	End of semester	50%

***Assessment task** (i.e., written test, oral test, oral presentation, group project, essay, etc.)

E. Student Academic Counseling and Support

Arrangements for availability of faculty and teaching staff for individual student consultations and academic advice :

Each group of students is assigned to a faculty member where he or she will provide academic advising. All faculty members are required to be in their offices outside teaching hours. Each faculty member allocates at least 4 hours per week to give academic advice and to answer to the questions of students about concepts studied during the lectures.

F. Learning Resources and Facilities

1. Learning Resources

Required Textbooks	1. An introduction to classical real analysis, (Karl, R. Stromberg) 2. Rudin, W.: Real and Complex Analysis, Third Edition, McGraw-Hill Book Company (1987).
Essential References Materials	1. A. Mukherjea and K. Pothoven, Real and Functional Analysis, Plenum Press, New York, 1978.

	2. Stein, E. M. and Shakarchi, R. Real Analysis - measure theory, integration and Hilbert spaces. (Princeton Lectures in Analysis III) Princeton University Press (2005).
Electronic Materials	http://ebookee.org/
Other Learning Materials	None

2. Facilities Required

Item	Resources
Accommodation (Classrooms, laboratories, demonstration rooms/labs, etc.)	Large classrooms that can accommodate more than 30 students.
Technology Resources (AV, data show, Smart Board, software, etc.)	Data Show.
Other Resources (Specify, e.g. if specific laboratory equipment is required, list requirements or attach a list)	None.

G. Course Quality Evaluation

Evaluation Areas/Issues	Evaluators	Evaluation Methods
Effectiveness of teaching and assessment.	Students	Direct
Quality of learning resources.	Students	Direct
Extent of achievement of course learning outcomes.	Faculty member	Direct

Evaluation areas (e.g., Effectiveness of teaching and assessment, Extent of achievement of course learning outcomes, Quality of learning resources, etc.)

Evaluators (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)

Assessment Methods (Direct, Indirect)

H. Specification Approval Data

Council / Committee	**Council of the Mathematics Department**	**The mathematical sciences (college of applied sciences) and the mathematics (Al-Leith University College) department's first meeting of the coordinative committee**
Reference No.	**4101050782**	**First meeting**
Date	**Sunday, 17 November 2019**	**Thursday, 17 October 2019**

Department Head

Dr. Ali Hassani